Problem 1

Write the program for numerical solution of the initial-boundary value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial}{\partial x}\left(D(u)\dfrac{\partial u}{\partial x}\right), \qquad x \in (0,\, 1), \quad t > 0, \\ u(0,t) = d_0(t), \\ u(1,t) = d_1(t), \\ u(x,0) = u_0(x), \end{cases} \tag{1}$$

for the nonlinear diffusion equation on the interval $[0,\, 1]$. The input data for the program are: a function $u_0 : [0,\, 1] \to \mathbb{R}$ and a concentration dependent diffusion coefficient $D(u) > 0$.

(a) Assume that the diffusion coefficient function is constant, e.g. $D(u) \equiv 1$, and $d_0(t) = d_0 = const$, $d_1(t) = d_1$. Check the results of the program with the results produced by the program written specifically for linear diffusion (labs 2 and 3). Especially confirm that for long time the solution tends to $(d_1 - d_0)x + d_0$.

(b) Solve numerically the problem (1) with diffusion coefficient $D(u) = \dfrac{1+2u^2}{1+u^2}$. Does it approach the solution of linear diffusion with $D = 1$ for long time?

Problem 2

Consider the simple reaction-diffusion problem (Fisher's model)

$$\begin{aligned} &\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \alpha u(1-u), \quad x \in (0,\, 1),\, t > 0, \\ &\frac{\partial u}{\partial x}(0,t) = 0,\, \frac{\partial u}{\partial x}(1,t) = 0, \\ &u(x,0) = u_0(x), \end{aligned} \tag{2}$$

where $u_0(x)$ is the initial profile assumed to have range in the unit interval ($0 \le u_0(x) \le 1$).

(a) Define an explicit finite difference scheme for numerical treatment of this problem.

(b) Write a program which computes the numerical solution to (2).

(c) Perform test of the code for the following data: $\alpha = 1,\, u_0(x) = \cos^2 \pi x$. Plot the profiles for $t = 0,\, 0.05,\, 05,\, 5$.

(d) How the numerical simulations behave if the stability condition for Fisher's equation is not met?