Problem 1

Write the program for numerical solution of the boundary value problem

$$\begin{cases} u'' = f(x,u,u'), \\ u(a) = d_0,\ u(b) = d_1, \end{cases}$$

on the interval $[a,\ b]$.

Test the code for the simple example of the linear equation

$$\begin{cases} u'' + 2u' + u = \sin x, \qquad x \in [0,\ 2\pi], \\ u(0) = 0,\ u(2\pi) = 0, \end{cases}$$

which has the following exact solution $u(x) = \left(\frac{1}{2} + \frac{e^{2\pi}-1}{4\pi}x\right)e^{-x} - \frac{1}{2}\cos x$. This could be solved by the program for the **Problem 4** from the **Problem set 02**, but this time we want to test our new code.

Problem 2

Solve numerically the problem

$$\begin{cases} u'' + e^{x}uu' = (-x^2 + 2x - 2)e^{-x}, \\ u(0) = 0,\ u(2) = -\dfrac{2}{e^2}, \end{cases}$$

and compare it with the exact solution $u(x) = xe^{-x}$. Test various grid sizes $h > 0$.

Problem 3

Consider the following (seemingly looking simply) problem

$$\begin{cases} u'' = u^2, \\ u(0) = 0,\ u(1) = 1. \end{cases}$$

Do the numerical simulations converge? How does the numerical solution look like? This problem is not trivial because the general theorem (remarked upon in footnote of the additional material) is not satisfied here (check it), so really we do not know whether we approximate the real solution or just some ghost solution.