Problem 1

Given the problem on the interval

$$\begin{cases} -(\lambda(x)u')' + \gamma(x)u = f(x), & x \in (a, b), \\ u(a) = g_0, \quad u(b) = g_1, \end{cases}$$

use the change o variable $\overline{x} = \dfrac{x-a}{b-a}$, for which $0 \le \overline{x} \le 1$, and covert the problem from the interval $[a, b]$ to the interval $[0, 1]$. Specifically, calculate the functions $\overline{\lambda}(\overline{x})$, $\overline{\gamma}(\overline{x})$, and $\overline{f}(\overline{x})$.

Problem 2

Using the codes from the course web page, or writing your own, perform simulations for the Dirichlet boundary value problem

$$\begin{cases} -((1+x^2)u')' + (\ln x)u = -2 + \ln x \ln(1+x^2), & x \in (1, 5), \\ u(1) = \ln 2, \; u(5) = \ln 26. \end{cases}$$

Compare the results with the exact solution $u(x) = \ln(1+x^2)$ for different grid size $h > 0$.

Problem 3

Write code for solving *mixed* Neumann-Dirichlet boundary value problem

$$\begin{cases} -u'' = f(x), \\ u'(a) = g_0, \; u(b) = d_1, \end{cases}$$

using two approaches for handling the Neumann boundary condition $u'(a) = g_0$: (i) ghost point method; (ii) three-points one sided difference.

Compare the results for different step sizes $h > 0$ on the model problem

$$\begin{cases} -u'' = f(x), & x \in (0, 2), \\ u'(0) = 1, \; u(2) = 0, \end{cases}$$

with exact solution $u(x) = (2-x)e^x$.

Problem 4

In the lab02 file we consider the general model problem of the second order boundary value ODE on the interval written in the so called divergent form

$$-(\lambda(x)u')' + \gamma(x)u = f(x), \quad x \in (a, b).$$

It is not always convenient to integrate equation in this form, and the following standard linear form

$$\begin{cases} u'' + p_1(x)u' + p_0(x)u = q(x), \\ u(a) = d_0,\ u(b) = d_1, \end{cases}$$

should also be considered.

Carry out the finite difference approximation, derive the proper system of linear equations for the approximate values, and implement procedure to solve the resulting tridiagonal linear system.

Perform simulation for the following problems

$$\begin{cases} u'' - 3u' + 2u = 1 + 2x, \\ u(1) = 0,\ \ u(2) = 0, \end{cases}$$

and

$$\begin{cases} xu'' + u = 0, \\ u(1) = 1,\ \ u(2) = 2. \end{cases}$$